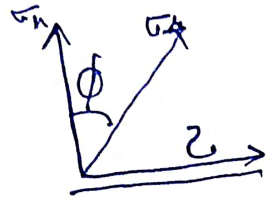
Compound Stresses & Strains

In many engineering problems, both direct (tensile or compressive) & shear stresses are brought in to play & the resultant stress across any section will be neither normal nor tangential to the plane. If σ_r is the resultant stress making an angle ϕ with the normal to the plane on which it acts



then $\qquad \phi = \tan^{-1}\dfrac{\tau}{\sigma_n}$

& $\qquad \sigma_r = \sqrt{\sigma_n^2 + \tau^2}$

→ **Methods for Determining Stresses On Oblique Section**

1. Analytical method, &
2. Graphical method

* <u>Analytical Method</u>

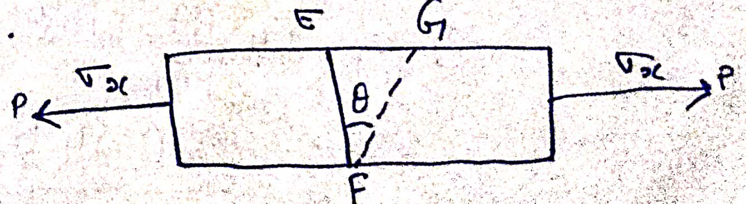
The following two cases will be considered.

1. A member subjected to a direct stress in one plane.
2. The member is subjected to like direct stress in two mutually perpendicular directions

→ 1. <u>Stresses On Oblique Plane Under Uniaxial Loading</u>

Consider a rectangular member of uniform cross-sectional area A & of unit thickness.

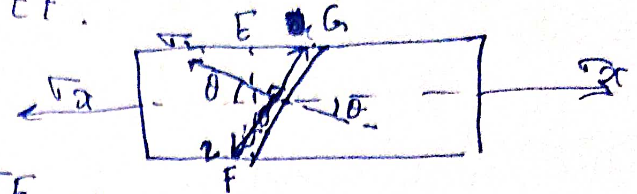
Let σ_x = Axial stress acting on the member



Consider a cross-section EF which is ⊥ to the line of action of the force P. The stress on the section EF is

(2) entirely normal stress. There is no shear stress or tangential stress on the section EF.

Now consider a section FG at an angle θ with the normal cross-section EF.



$\therefore$ Area of section $FG = \frac{EF}{\cos\theta} \times 1$

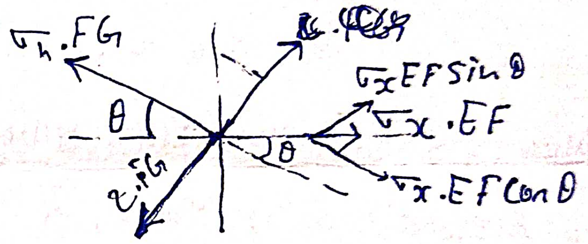
$= A \sec\theta$

Let σ_n = Normal stress across the section FG

τ = Tangential or shear stress across section FG

Resolving the forces $\perp$ to the plane FG

$$\sigma_n . FG = \sigma_x . EF \cos\theta$$



$$\sigma_n = \sigma_x \frac{EF}{FG} \cos\theta$$

$$\sigma_n = \sigma_x \cos^2\theta$$

$$\boxed{\sigma_n = \frac{\sigma_x}{2}(1+\cos 2\theta)}$$

The normal stress on plane FG is maximum or minimum according as $\cos 2\theta$ is max. or mini.

For $\theta = 0^\circ$, $\sigma_n = \sigma_x$

& for $\theta = 90^\circ$, $\sigma_n = 0$

Now resolving forces $\parallel$ to plane FG.

$$\tau . FG = +\sigma_x EF \sin\theta$$

$$\tau = \sigma_x \frac{EF}{FG} \sin\theta$$

$$= \sigma_x \cos\theta \sin\theta$$

$$\boxed{\tau = \frac{\sigma_x}{2} \sin 2\theta}$$

Resultant stress $\sigma_r = \sqrt{\sigma_n^2 + \tau^2}$

$$= \frac{\sigma_x}{2}\left[(1+\cos 2\theta)^2 + \sin^2 2\theta\right]^{\frac{1}{2}}$$

$$= \sigma_x \left[\frac{1+\cos 2\theta}{2}\right]^{1/2} = \sigma_x \cos\theta$$

If the resultant stress makes an angle ϕ with normal stress then,

$$\tan\phi = \tau/\sigma_n = \frac{\sigma_x \sin\theta \cos\theta}{\sigma_x \cos^2\theta}$$

$$= \tan\theta$$

$$\therefore \phi = \theta$$

Now, Shear stress $\tau = \frac{\sigma_x}{2}\sin 2\theta$

τ is max when $\sin 2\theta$ is max. i.e.

$$\sin 2\theta = \pm 1$$

$$2\theta = 90^\circ \text{ or } 270^\circ$$

$$\theta = 45^\circ \text{ or } 135^\circ$$

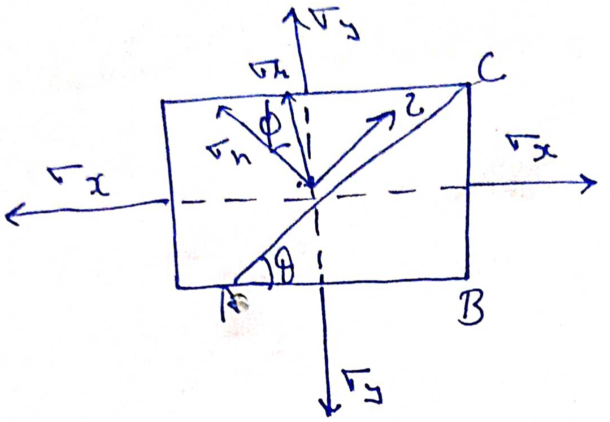
$$\therefore \boxed{\tau_{max} = \frac{\sigma_x}{2}}$$

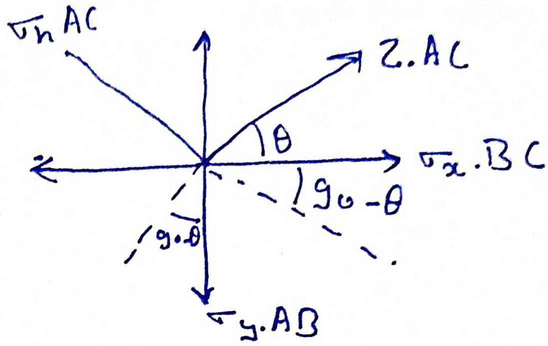
Therefore for a state of uniaxial stress the max. tangential stress occurs along planes, the normal to which make angle of 45° & 135° with the direction of the load.

Conclusion :- If a material is such that its shear strength is less than half of its tensile strength, then the material will fail by shear.

2. STESSES ON AN OBLIQUE PLANE UNDER BIAXIAL LOADING

(a) Like stress :- Consider a body under the action of biaxial stresses σ_x & σ_y. Let AC be the oblique plane inclined at an angle θ with the plane AB which is ∥ to the line of action of σ_x. Let σ_n & τ be





the normal & shear stresses on plane AC.

Now, By resolving the forces ⊥ to plane AC

$$\sigma_n \times AC = \sigma_x \cdot BC \cdot \sin\theta + \sigma_y \cdot AB \cdot \cos\theta$$

$$\sigma_n = \sigma_x \cdot \frac{BC}{AC} \cdot \sin\theta + \sigma_y \cdot \frac{AB}{AC} \cdot \cos\theta$$

$$= \sigma_x \sin^2\theta + \sigma_y \cos^2\theta$$

$$= \sigma_x \left(\frac{1-\cos 2\theta}{2}\right) + \sigma_y \left(\frac{1+\cos 2\theta}{2}\right)$$

$$\boxed{\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_y - \sigma_x}{2}\right) \cos 2\theta}$$

Similarly, by resolving the forces ∥ to plane AC;

$$\tau \times AC = -\sigma_x \cdot BC \cos\theta + \sigma_y \cdot AB \sin\theta$$

$$\tau = -\sigma_x \cdot \frac{BC}{AC} \cos\theta + \sigma_y \frac{AB}{AC} \sin\theta$$

$$= -\sigma_x \sin\theta \cos\theta + \sigma_y \cos\theta \sin\theta$$

$$= (\sigma_y - \sigma_x) \sin\theta \cos\theta$$

$$\boxed{\tau = \frac{1}{2}(\sigma_y - \sigma_x) \sin 2\theta}$$

∴ The shear stress is max. when $\theta = 45°$ & its max. value is

$$\tau_{max} = \frac{1}{2}(\sigma_y - \sigma_x)$$

At $\theta = 45$, the value of normal stress is,

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2}$$

RESULTANT STRESS $\sigma_r = \sqrt{\sigma_n^2 + \tau^2}$

$$= \left[\left\{\frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_y-\sigma_x}{2}\cos 2\theta\right\}^2 + \left(\frac{\sigma_y-\sigma_x}{2}\right)^2 \sin^2 2\theta\right]^{1/2}$$

$$= \left[\left(\frac{\sigma_x+\sigma_y}{2}\right)^2 + \left(\frac{\sigma_y-\sigma_x}{2}\right)^2\cos^2 2\theta + 2 \cdot \frac{\sigma_x+\sigma_y}{2} \cdot \frac{\sigma_y-\sigma_x}{2} \cdot \cos 2\theta + \left(\frac{\sigma_y-\sigma_x}{2}\right)^2 \sin^2 2\theta\right]^{1/2}$$

$$= \left[\left(\frac{\sigma_x+\sigma_y}{2}\right)^2 + \frac{1}{2}(\sigma_x+\sigma_y)(\sigma_y-\sigma_x)\cos 2\theta + \left(\frac{\sigma_y-\sigma_x}{2}\right)^2(\cos^2 2\theta + \sin^2 2\theta)\right]^{1/2}$$

$$= \left[\frac{\sigma_x^2}{4} + \frac{\sigma_y^2}{4} + \cancel{\frac{2\sigma_x\sigma_y}{4}} + \frac{\sigma_y^2}{4} + \frac{\sigma_x^2}{4} - \cancel{\frac{2\sigma_y\sigma_x}{4}} + \frac{1}{2}(\sigma_y^2 - \sigma_x^2)\cos 2\theta\right]^{1/2}$$

$$= \left[\frac{1}{2}(\sigma_x^2 + \sigma_y^2) + \frac{1}{2}(\sigma_y^2 - \sigma_x^2)\cos 2\theta\right]^{1/2}$$

If σ_r is inclined at an angle ϕ with σ_n, then

$$\tan\phi = \frac{\tau}{\sigma_n} = \frac{(\sigma_y - \sigma_x)\sin\theta\cos\theta}{\sigma_x \sin^2\theta + \sigma_y\cos^2\theta}$$

$$\Rightarrow \tan\phi = \frac{\sigma_y - \sigma_x}{\sigma_x\tan\theta + \sigma_y\cot\theta}$$

$$\phi = \tan^{-1}\left[\frac{\sigma_y - \sigma_x}{\sigma_x\tan\theta + \sigma_y\cot\theta}\right]$$

For greatest obliquity, $\frac{d(\tan\phi)}{d\theta} = 0$

$$\therefore \sigma_x\sec^2\theta - \sigma_y\operatorname{cosec}^2\theta = 0$$

$$\Rightarrow \boxed{\tan\theta = \sqrt{\frac{\sigma_y}{\sigma_x}}} \quad \& \quad \boxed{\tan\phi_{max} = \frac{\sigma_y - \sigma_x}{2\sqrt{\sigma_x\sigma_y}}}$$

(b) ~~W~~ Unlike stresses :- let σ_x be tensile & σ_y compressive in nature, then

Take tensile as +ve
Compressive as -ve

$$\therefore \sigma_n = \sigma_x \sin^2\theta - \sigma_y \cos^2\theta$$

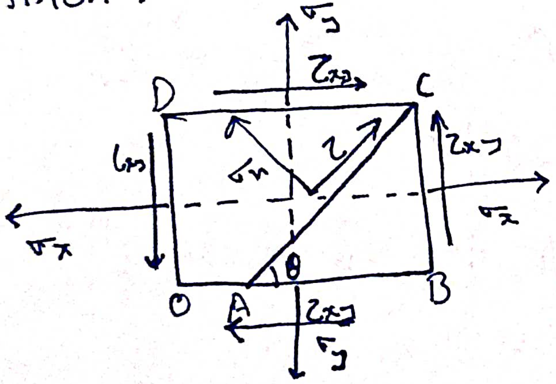
$$= \frac{1}{2}(\sigma_x - \sigma_y) - \frac{1}{2}(\sigma_x + \sigma_y)\cos 2\theta$$

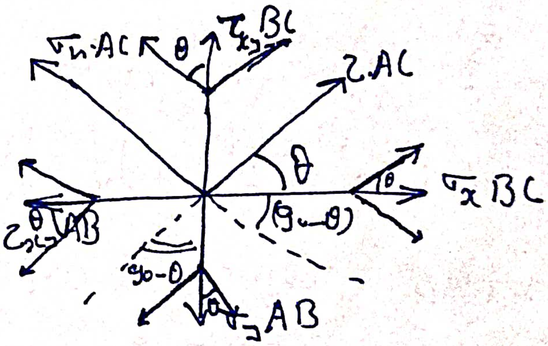
$$\& \ \tau = -\frac{1}{2}(\sigma_x + \sigma_y)\sin 2\theta$$

$$\& \ \tau_{max} = -\frac{1}{2}(\sigma_x + \sigma_y)$$

2(b) BIAXIAL STRESSES COMBINED With SHEAR STRESSES

Consider a rectangular body OBCD of unit thickness subjected to direct stress σ_x & σ_y & shear stress τ_{xy}. Let AC be the oblique plane making an angle θ with the σ_x stress.





Let σ_n & τ be the normal & shear stress respectively acting on the plane AC.

Now by resolving all the forces along the direction of σ_n, we get

$$\sigma_n AC + \tau_{xy} BC \cos\theta - \sigma_x BC \sin\theta - \sigma_y AB \cos\theta + \tau_{xy} AB \sin\theta = 0$$

$$\Rightarrow \sigma_n = \sigma_x \frac{BC}{AC}\sin\theta + \sigma_y \frac{AB}{AC}\cos\theta - \tau_{xy}\frac{BC}{AC}\cos\theta - \tau_{xy}\frac{AB}{AC}\sin\theta$$

$$= \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy}\sin\theta\cos\theta$$

$$= \sigma_x\left(\frac{1-\cos 2\theta}{2}\right) + \sigma_y\left(\frac{1+\cos 2\theta}{2}\right) - \tau_{xy}\sin 2\theta$$

$$\boxed{\sigma_n = \tfrac{1}{2}(\sigma_y + \sigma_x) + \tfrac{1}{2}(\sigma_y - \sigma_x)\cos 2\theta - \tau_{xy}\sin 2\theta}$$

Now resolving all the forces ∥ to plane AC

$$\tau . AC + \sigma_x BC \cos\theta - \sigma_y AB \sin\theta - \tau_{xy} AB \cos\theta + \tau_{xy} BC \sin\theta = 0$$

$$\Rightarrow \tau = \sigma_y \frac{AB}{AC}\sin\theta - \sigma_x \frac{BC}{AC}\cos\theta + \tau_{xy}\frac{AB}{AC}\cos\theta - \tau_{xy}\frac{BC}{AC}\sin\theta$$

$$= (\sigma_y - \sigma_x)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$$

$$\boxed{\tau = \tfrac{1}{2}(\sigma_y - \sigma_x)\sin 2\theta + \tau_{xy}\cos 2\theta}$$

For σ_n to have max. or mini. value

$$\frac{d\sigma_n}{d\theta} = 0$$

$$\Rightarrow \frac{d\left[\tfrac{1}{2}(\sigma_y + \sigma_x) + \tfrac{1}{2}(\sigma_y - \sigma_x)\cos 2\theta - \tau_{xy}\sin 2\theta\right]}{d\theta} = 0$$

$$\Rightarrow -(\sigma_y - \sigma_x)\sin 2\theta - 2\tau_{xy}\cos 2\theta = 0$$

$$\Rightarrow \tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \quad \boxed{1}$$

This gives the two values of θ

Note:- Also by putting $\tau = 0$, we can find $\tan 2\theta = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$
& while resolving the forces, take $\sigma_x BC$ & $\sigma_y AB$ in rightward & downward direction & $\tau_{xy} BC$ & $\tau_{xy} AB$ in upward & leftward direction respectively or you can take vice versa.

Now for the shear stress τ has a max. or mini value, $\dfrac{d\tau}{d\theta} = 0$

$$\Rightarrow \frac{d\left[\tfrac{1}{2}(\sigma_y - \sigma_x)\sin 2\theta + \tau_{xy}\cos 2\theta\right]}{d\theta} = 0$$

$$\Rightarrow \tan 2\theta = \frac{\sigma_y - \sigma_x}{2\tau_{xy}} = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right) \quad (2)$$

The two values of 2θ differ by $180°$ &

hence two values of θ differing by $90°$.

Note :- Comparing eqn. (1) & (2)

$$\tan 2\theta = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\frac{1}{\tan 2\theta_1}$$

$$\Rightarrow \tan 2\theta = -\cot 2\theta_1$$

$$\Rightarrow \tan 2\theta = \tan(90 + 2\theta_1)$$

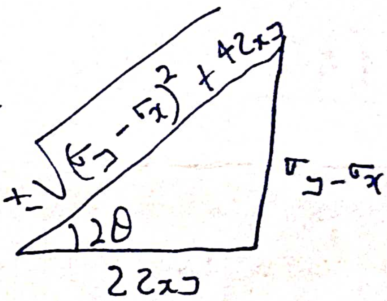
$$\Rightarrow 2\theta = 90 + 2\theta_1$$

$$\Rightarrow \theta = 45° + \theta_1$$

∴ The plane of principal stress are inclined at 45° to the plane of max. shear stress.

* To find τ_{max}

$$\tau_{max} = -\frac{1}{2}\frac{(\sigma_y - \sigma_x)(\sigma_x - \sigma_y)}{\pm\sqrt{(\sigma_y-\sigma_x)^2 + 4\tau_{xy}^2}} + \frac{\tau_{xy} 2\tau_{xy}}{\pm\sqrt{(\sigma_y-\sigma_x)^2 + 4\tau_{xy}^2}}$$



$$\because \tau = \frac{1}{2}(\sigma_y - \sigma_x)\sin 2\theta + \tau_{xy}\cos 2\theta$$

& from rt. ∠ triangle

$$\sin 2\theta = \frac{\sigma_y - \sigma_x}{\pm\sqrt{(\sigma_y-\sigma_x)^2 + 4\tau_{xy}}}, \quad \cos 2\theta = \frac{2\tau_{xy}}{\pm\sqrt{(\sigma_y-\sigma_x)^2 + 4\tau_{xy}}}$$

$$\therefore \tau_{max} = \frac{1}{2}\left\{\frac{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}{\pm\sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}}\right\}$$

$$\boxed{\tau_{max} = \pm\frac{1}{2}\{(\sigma_x - \sigma_x)^2 + 4\tau_{xy}^2\}^{1/2}}$$

or $\boxed{\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_2)}$

PRINCIPAL STESSES & PRINCIPAL PLANES

PRINCIPAL PLANES :- With in a stressed body at any point, there always exist three mutually perpendicular planes on each of which the resultant stress is a normal stress. There is no shear stress. These mutually perpendicular planes are called principal planes.

PRINCIPAL STRESSES :- The resultant normal stresses acting on the principal planes are called principal stresses. In the case of two dimensional problems, one of the principal stress is zero & out of other two, one is the greatest & the other is the least stress.

Position of principal planes :-

Shear stress, in case of biaxial stresses combined with shear stresses is given by

$$\tau = \frac{1}{2}(\sigma_y - \sigma_x)\sin 2\theta + \tau_{xy}\cos 2\theta$$

Put $\tau = 0$

$$\therefore \frac{1}{2}(\sigma_y - \sigma_x)\sin 2\theta + \tau_{xy}\cos 2\theta = 0$$

$$\Rightarrow \frac{\sin 2\theta}{\cos 2\theta} = \frac{-2\tau_{xy}}{\sigma_y - \sigma_x}$$

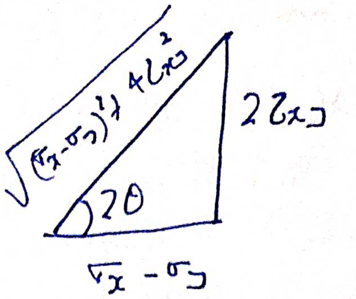
$$\Rightarrow \boxed{\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}}$$

This gives two values of 2θ differing by $180°$ & hence two values of θ differing by $90°$, ie the principal planes are two planes at right angle.

Magnitude of Principal stresses :-

from $\tan 2\theta = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

make a right angle triangle as shown & find hypotenuse



$$\therefore \sin 2\theta = \frac{2\tau_{xy}}{\pm\sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}}$$

$$\& \cos 2\theta = \frac{\sigma_x - \sigma_y}{\pm\sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}}$$

Normal stress is given by relation

$$\sigma_n = \tfrac{1}{2}(\sigma_x + \sigma_y) + \tfrac{1}{2}(\sigma_y - \sigma_x)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_n = \tfrac{1}{2}(\sigma_x+\sigma_y) + \tfrac{1}{2}(\sigma_y - \sigma_x)\frac{\sigma_x - \sigma_y}{\pm\sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}} - \tau_{xy}\frac{2\tau_{xy}}{\pm\sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}}$$

$$= \tfrac{1}{2}(\sigma_x + \sigma_y) - \tfrac{1}{2}\left\{\frac{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}{\pm\sqrt{(\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2}}\right\}$$

or $\sigma_1, \sigma_2 = \tfrac{1}{2}(\sigma_x - \sigma_y) \pm \tfrac{1}{2}\{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2\}^{1/2}$

This gives two values of σ_n, one of which is max. & the other minimum which are the two principal stresses σ_1 & σ_2 respectively.

MOHR'S CIRCLE :-

It is a graphical method of finding normal, tangential & resultant stresses on an oblique plane.

Mohr's circle will be drawn for the following cases:

(i) A body subjected to two mutually perpendicular principal tensile stresses of unequal intensities.

(ii) A body subjected to two mutually perpendicular principal stresses which are unequal & unlike (i.e., one is tensile & other is compressive)

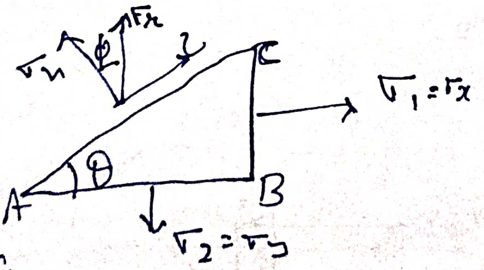
(iii) A body subjected to two mutually ⊥ principal stresses accompained by a simple shear stress.

Ist case(i):-

Let σ_1 = Major tensile stress (or principal stress)

σ_2 = Minor tensile stress (or minor principal stress)

On the principal plane BC & AB. The stress circle will be developed to find the stress components on any plane AC which makes angle θ with AB.

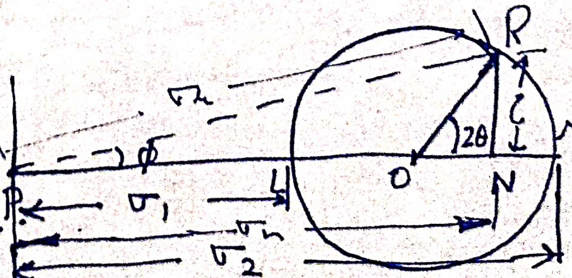


METHOD TO MAKE MOHR'S CIRCLE :-

Mark off $PL = \sigma_1 = \sigma_x$ $\quad \sigma_2 > \sigma_1$ or $\sigma_y > \sigma_x$

$PM = \sigma_2 = \sigma_y$

On LM as diameter describe a circle centre O.



Then the radius OL 'represents' the plane of σ_1 (BC), & OM 'represents' the plane of σ_2 (AB). Plane AC is obtained by rotating AB through θ anticlockwise, & if OM on the stress circle is rotated through 2θ in the same direction, the radius OR is obtained which will again be shown to represent the plane AC.

Draw RN ⊥ PM

Then $PN = PO + ON$

$$= \frac{1}{2}(\sigma_1+\sigma_2) + \frac{1}{2}(\sigma_2-\sigma_1)\cos 2\theta$$

$$= \sigma_1\left(\frac{1-\cos 2\theta}{2}\right) + \sigma_2\left(\frac{1+\cos 2\theta}{2}\right)$$

$$= \sigma_1 \sin^2\theta + \sigma_2 \cos^2\theta$$

$= \sigma_n$, the normal stress component on AC.

& $RN = \frac{1}{2}(\sigma_2-\sigma_1)\sin 2\theta$

$= \tau$, the shear stress component on AC.

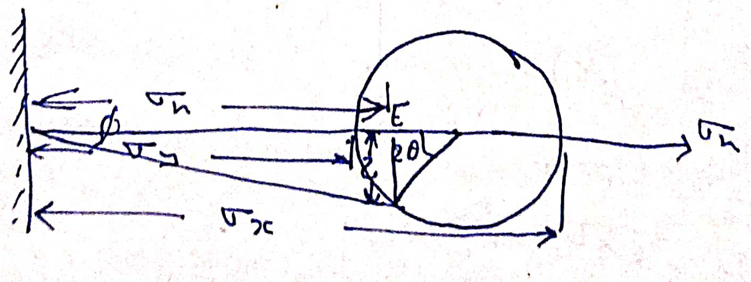
Also the resultant stress

$$\sigma_r = \sqrt{\sigma_n^2+\tau^2}$$

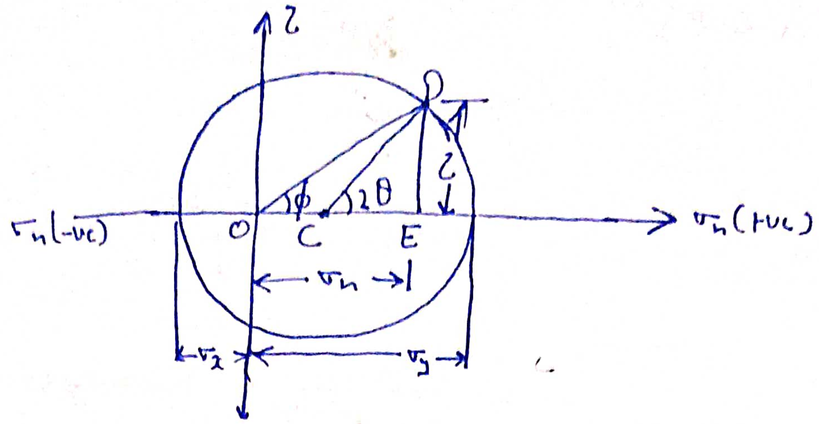
& its inclination to the normal of the plane is given by

$\phi = \angle RPN$.

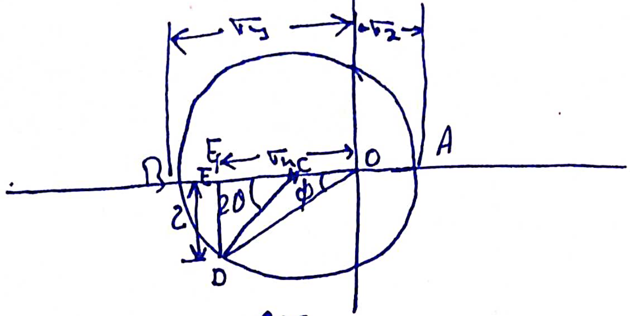
Other case :- $\sigma_x > \sigma_y$



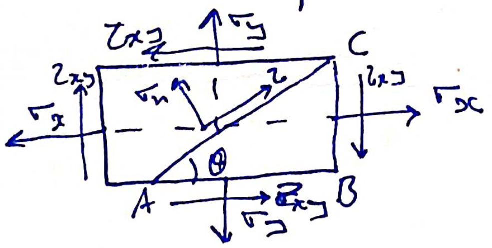
case (ii) (a) σ_x is compressive



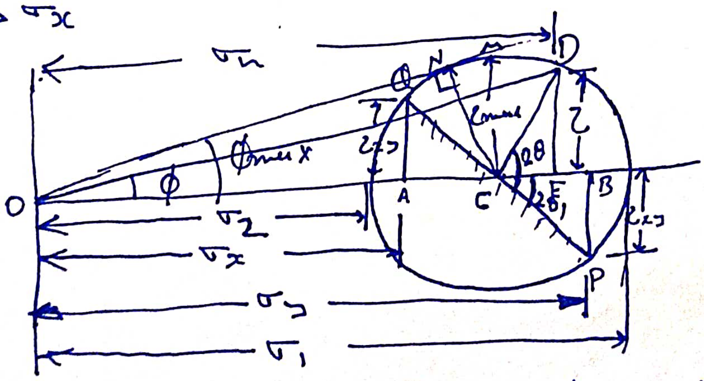
(b) σ_y is compressive



Case (iii)



(a) $\sigma_y > \sigma_x$



Mark off $OA = \sigma_x$, $OB = \sigma_y$, & take $AQ = BP = \tau_{xy}$

→ Join PQ.

→ With C as centre & CP as radius draw the circle.

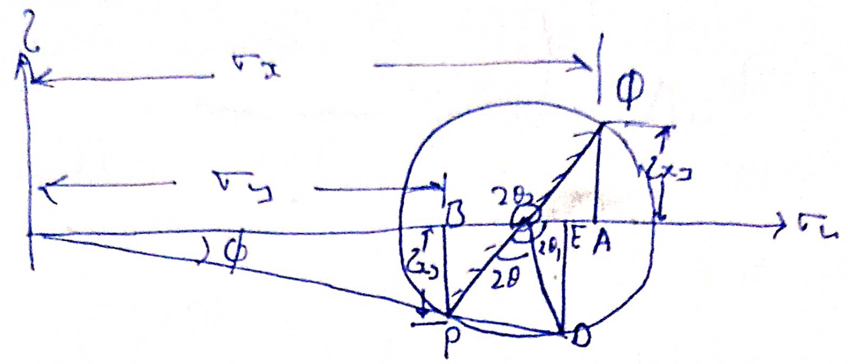
→ Draw $\angle PCD = 2\theta$

→ Draw $DE \perp AB$ & Join OD.

Then $OE = \sigma_n$, $DE = \tau$, $OD = \sigma_r$, $OG = \sigma_1$, $\angle DOC = \phi$

$OF = \sigma_2$, $\angle PCG = 2\theta_1$, $\angle PCF = 2\theta_2$, $\angle NOF = \phi_{max}$

(b) $\sigma_y < \sigma_x$



MOHR'S CIRCLE :-

It is a graphical method of finding normal, tangential & resultant stress on an oblique plane.

Mohr's circle will be drawn for following cases generally:

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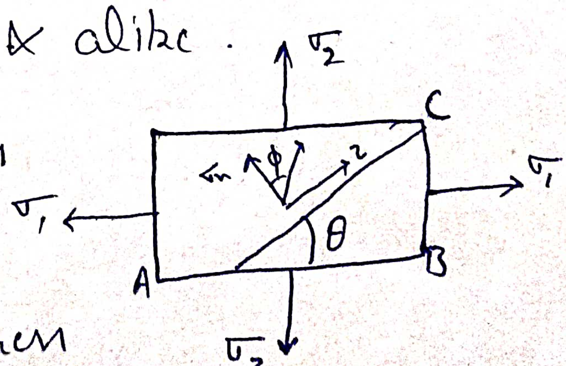
(ii) A body subjected to two mutually perpendicular principal stresses which are unequal & unlike (i.e. one is tensile & other is compressive)

(iii) A body subjected to two mutually perpendicular principal stresses accompanied by simple shear stress.

Case (i) :- A body is subjected to two mutually perpendicular principal tensile stresses of unequal intensities & alike.



Given:-

Let, σ_2 = Major tensile stress or Max. principal stress

& σ_1 = Minor tensile stress or Minimum principal stress

θ = Angle made by oblique plane with minor tensile stress.

To find out

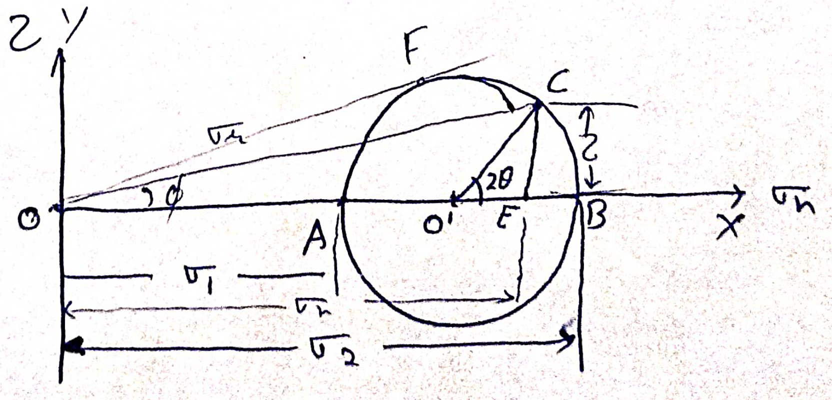
Normal stress on oblique plane $= \sigma_n$
Shear stress on oblique plane $= \tau$
Resultant stress of σ_n & τ $= \sigma_R$
Resultant angle of σ_R with σ_n $= \phi$
Max. shear stress $= \tau_{max}$
Max. resultant angle $= \phi_{max}$

How to make Mohr's circle of stress with given data & to find out stresses on oblique plane

Step 1:- Take origin O & draw horizontal line & vertical line which passes through it i.e. OX & OY.

NOTE:- All shear stresses taken on vertical line & direct stresses on horizontal line.

Step 2:- Cut OA & OB equal to σ_1 & σ_2 respectively by taking suitable scale.



Step 3:- Bisect AB at O' & take O' as centre & O'A as radius, draw circle

Step 4 :- At centre O', draw an inclined line at an angle of 2θ which cuts the circle at point C

Step 5 :- Through point C, draw a perpendicular line to the horizontal line which cuts it at point E. Also join OC.

Step 6 :- Now, Measure OC, OE & CE. These are equivalent to σ_R, σ_n, & τ respectively.

Step 7 :- Measure ∠COB, O'C (radius of circle) also.

$$\angle COB \sim \phi$$
$$O'C \sim \tau_{max}$$

Step 8 :- For finding out ϕ_{max}, draw a line from O', which is tangential to the circle cuts circle at F. $\angle FOB \sim \phi_{max}$

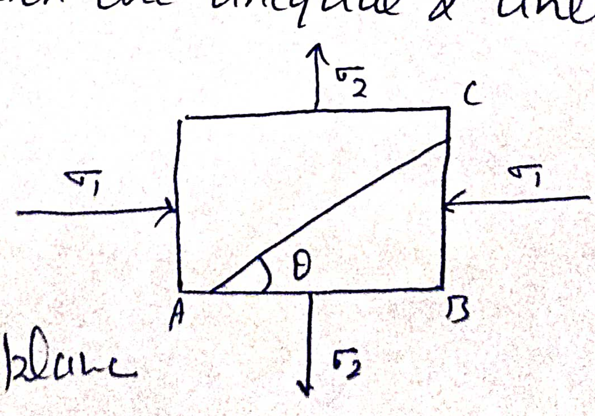
Case (ii) A body is subjected to two mutually perpendicular principal stresses which are unequal & unlike.

Given

σ_1 = Compressive stress

σ_2 : Tensile stress

θ = Angle of oblique plane



To find out

Normal stress on oblique plane, σ_n

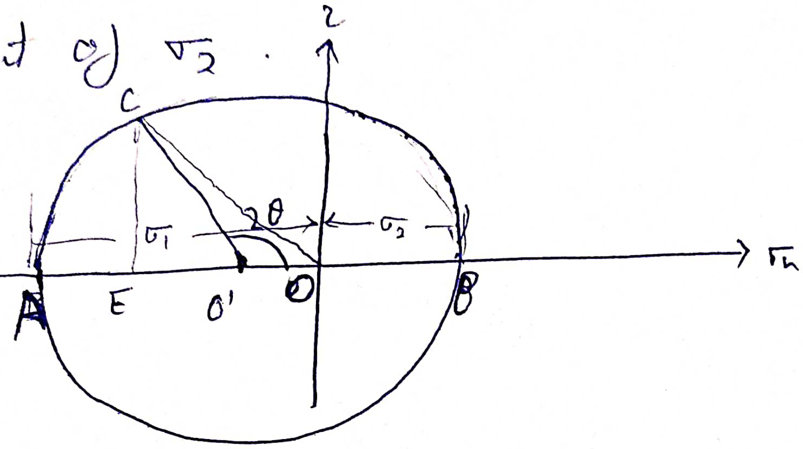
Shear stress on oblique plane, τ

Resultant stress of σ_n & τ

Angle of obliquity of resultant stress ϕ

Step 1 :- Take origin O & draw horizontal line & vertical line which passes through it i.e. OX & OX.

Step 2 :- Cut OA & OB equal to σ_1 & σ_2 respectively by taking suitable scale. σ_1 is compressive in nature, so it is taken on opposite side to that of σ_2.



Step 3 :- Bisect AB at O' & take O' as centre & O'A as radius, draw a circle.

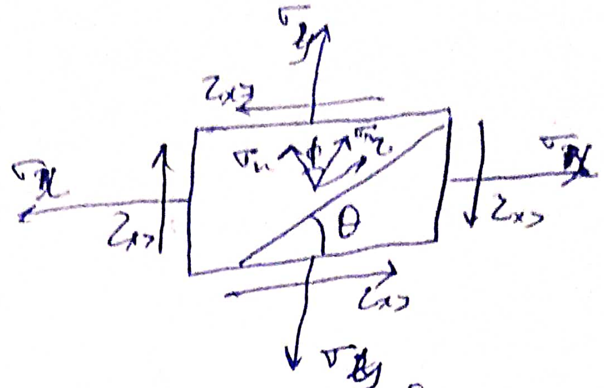
Step 4 :- At centre O', draw an inclined line at an angle of 2θ which cut the circle at point C.

Step 5 :- Through point C, draw a perpendicular line to the horizontal line which cuts it at point E. Also join OC.

Step 6 :- Now, measure OC, OE, CE & $\angle COA$. These are equivalent to σ_r, σ_n, z & ϕ respectively. Convert these in to stress by multiplying with assumed scale.

Case (iii) A body is subjected to two mutually perpendicular principal stresses accompanied by simple shear stress

(a) $\sigma_y > \sigma_x$



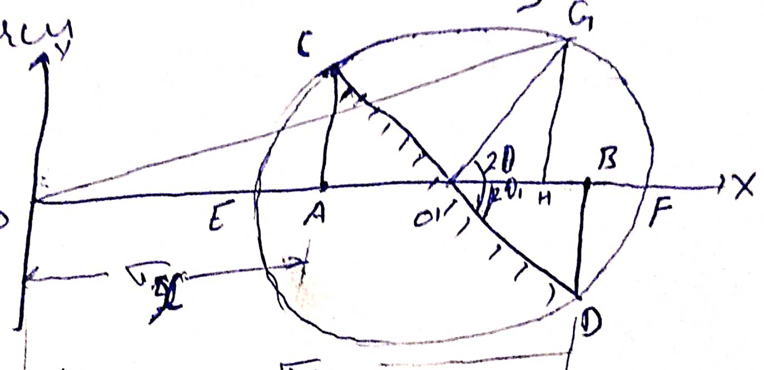
Given :-

σ_y = Major tensile stress

σ_x = Minor tensile stress

τ_{xy} = Shear stress

θ = Angle made by oblique plane with minor tensile stress



To find out :-

Maximum & Minimum Principal stresses i.e. σ_1 & σ_2

Direction of plane of σ_1 & σ_2 i.e. θ_1, θ_2 i.e. Principal planes

Maximum shear stress i.e. τ_{max}

Direction of planes of maximum shear stress i.e. θ_3 & θ_4

Normal stress on oblique plane i.e. σ_n

Shear stress on oblique plane i.e. τ

Resultant stress of σ_n & τ i.e. σ_R

Resultant angle of σ_R & σ_n i.e. ϕ

Sol.

Step I

1. Mark off $OA \sim \sigma_x$, $OB \sim \sigma_y$ on OX line
2. Also take AC & BD in opposite direction equivalent to τ_{xy}
3. Join CD which cuts the horizontal line at O'.

(4) With O' as centre & O'C as radius, draw circle which cuts the horizontal line at E & F.

5. Draw an angle of 2θ with CD as base line & O' as centre i.e. ∠GO'D = 2θ.

6. Draw perpendicular from point G on OX ~~line~~ line which cuts it at H. Join OG also

7. Now measure OE & OF, which equivalent to σ_2 & σ_1.

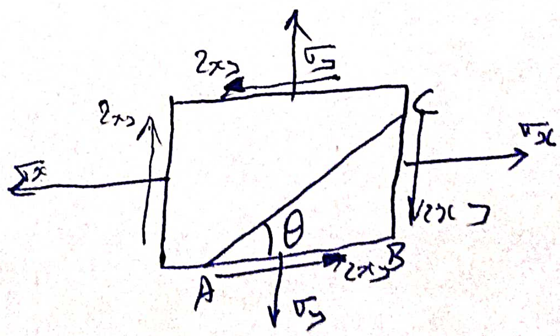
Measure ∠BO'D ~ $2\theta_1$, & $\theta_3 = \theta_1 + 45^\circ$

$\theta_2 = \theta_1 + 90^\circ$ $\theta_4 = \theta_2 + 45^\circ$

Measure GH, OG, OH, ∠GOO' which are equivalent to τ, σ_R, σ_n & ϕ respectively

Convert it by multiplying by assumed scale.

(b) $\sigma_x > \sigma_y$



Given:-

σ_x = Major tensile stress

σ_y = Minor tensile stress

τ_{xy} = Shear stress

θ = Angle made by oblique plane with major tensile stress

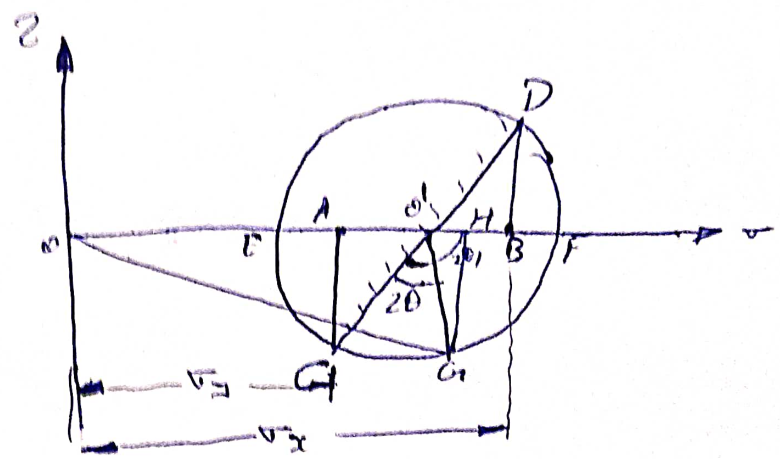
To find out :- Max. & Min. Principal stresses i.e. σ_1 & σ_2

Direction of plane of σ_1 & σ_2 i.e. θ_1 & θ_2

Max shear stress i.e. τ_{max}

Direction of planes of max. shear stress i.e. θ_3 & θ_4
Normal stress on oblique plane i.e. σ_n
Shear stress on oblique plane i.e. τ
Resultant stress of σ_n & τ i.e. σ_R
Resultant angle of σ_R & σ_n i.e. ϕ

Sol.



Steps
① Mark off $OA = \sigma_y$, $OB = \sigma_x$ on horizontal line by assuming suitable scale.
② Also take AC & BD in opposite direction equivalent to τ_{xy}.
Note:- AC in downward direction & BD in upward direction where as in previous case AC ~~was~~ in upward direction & BD in downward direction.
③ Join CD which cuts the horizontal line at O'.
④. With O' as centre & O'C as radius, draw circle which cuts the horizontal line at E & F.
⑤. Draw an angle of 2θ with CD as base line & O' as centre i.e. $\angle GO'C = 2\theta$ in anticlockwise direction.

(6) ∴ Draw perpendicular from point G on OX line which cuts it at H. Join OG also.

(7) Now measure OE & OF, which equivalent to σ_2 & σ_1

Measure $\angle BO'C = 2\theta_1$

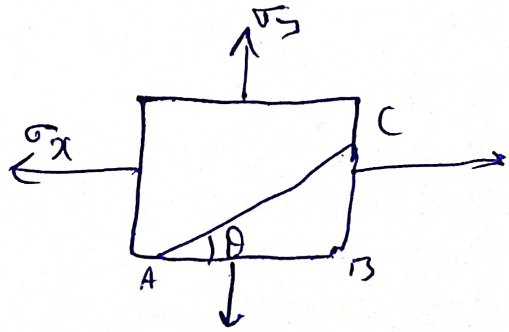
$$\theta_1 = \frac{\angle BO'C}{2} \quad \& \quad \theta_3 = \theta_1 + 45^\circ$$

$$\theta_2 = \theta_1 + 90^\circ \qquad \theta_4 = \theta_2 + 45^\circ$$

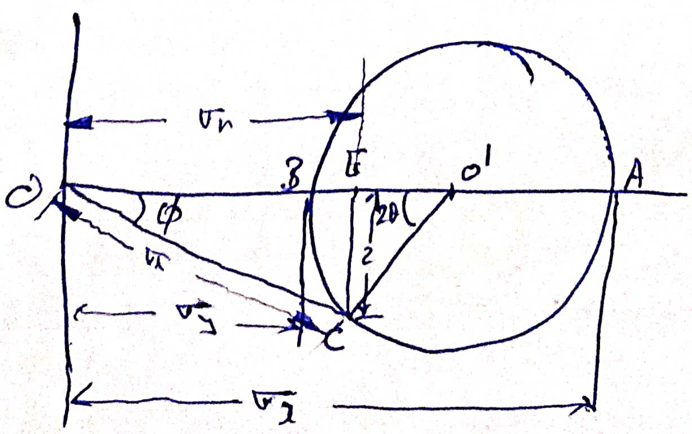
Measure GH, OG, OH, $\angle GOO'$ which are equivalent to τ, σ_r, σ_n & ϕ respectively. Convert it by multiplying by assumed scale.

OTHER CASES

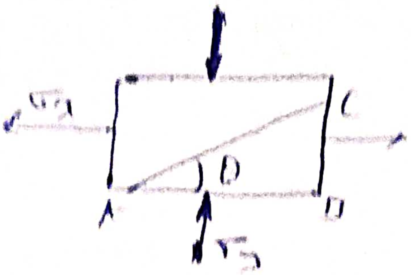
(i) $\sigma_y < \sigma_x$, when a body is subjected to two mutually perpendicular stresses only

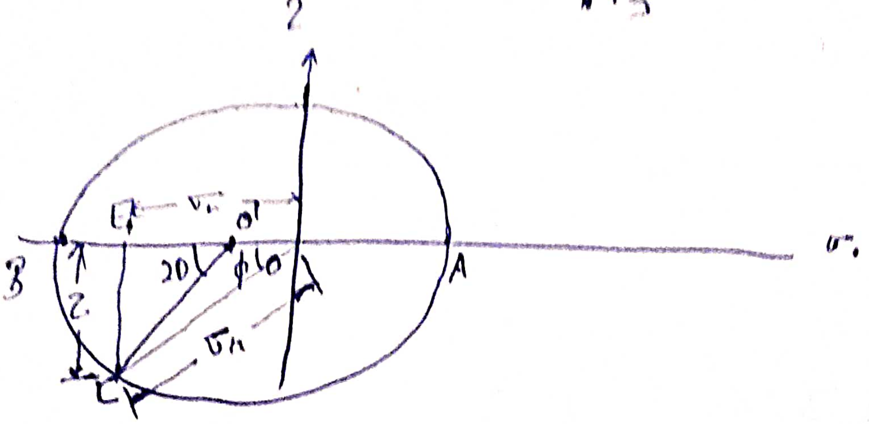


Sol.



(iii.) σ_y is compressive, when a body is subjected to two mutually perpendicular stresses only.





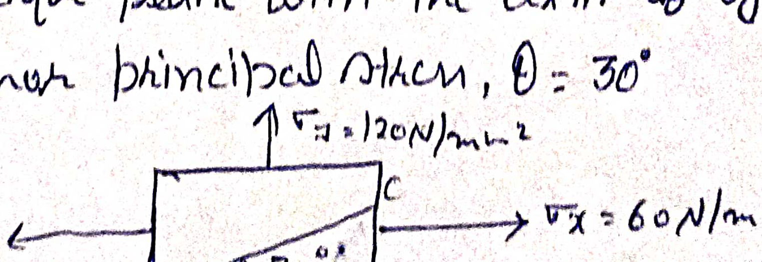
Problem 1.10

The principal tensile stresses at a point across two mutually perpendicular planes are $120\,N/mm^2$ & $60\,N/mm^2$. Determine the normal, tangential & resultant stresses on a plane inclined at 30° to the axis of the minor principal stress by Mohr's circle method.

Sol. :- Given Major principal stress, $\sigma_y = 120\,N/mm^2$ (tensile)

Minor principal stress, $\sigma_x = 60\,N/mm^2$ (tensile)

Angle of oblique plane with the axis of minor principal stress, $\theta = 30^\circ$

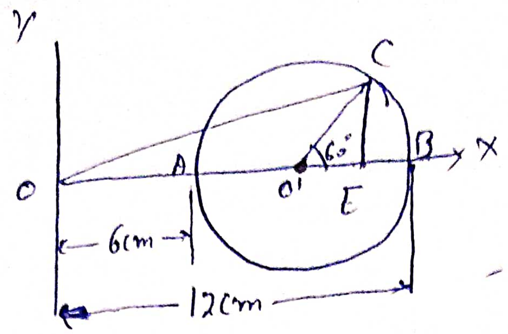


Let us assume a scale $1\,cm = 10\,N/mm^2$

$\therefore \sigma_y = 120\,N/mm^2 \simeq 12\,cm$

& $\sigma_x = 60\,N/mm^2 \simeq 6\,cm$

∴ Mohr's circle is drawn as:



1. Take origin O & draw vertical line OY & horizontal line OX.
2. Take OA = σ_x = 6cm & OB = σ_y = 12cm
3. Bisect AB at O' & with AB as diameter, draw circle.
4. At centre O', draw an inclined line with angle $2\theta = 60°$ which cuts the circle at point C.
5. Draw CE ⊥ OX & Join OC also
6. Measure OC, OE, CE & ∠COB

By Measurements:

Length OE = 10.5cm & ∠COE = ϕ = Ans

CE = 2.60 cm

OC = 10.82 cm

∴ Normal stress, σ_n = OE × scale

= 10.5 × 10 = 105 N/mm² Ans

Tangential or shear stress, τ = CE × scale

= 2.6 × 10 = 26 N/mm² Ans

& Resultant stress, σ_R = 10.82 × 10 = 108.2 N/mm² Ans

Problem 1.11

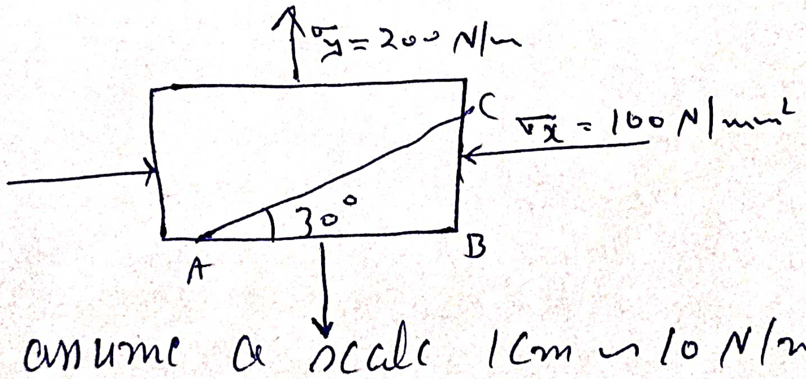
The principal stresses at a point in a bar are 200 N/mm^2 (tensile) & 100 N/mm^2 (compressive). Determine the resultant stress in magnitude & direction on a plane inclined at 30° to the axis of the minor principal stress (or to the plane of major principal stress i.e. major principal plane). Also determine the maximum intensity of shear stress in a material at that point.

Sol. Given Major Principal stress, $\sigma_y = 200\,N/mm^2$ (tensile)

Minor Principal stress, $\sigma_x = 100\,N/mm^2$ (comp.)

i.e. $-100\,N/mm^2$

Angle of oblique plane with the axis of minor principal stress, $\theta = 30°$

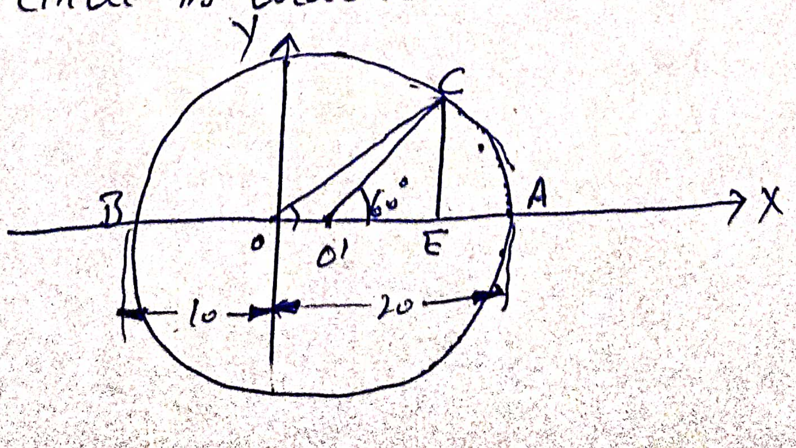


Let us assume a scale 1 cm ~ 10 N/mm^2

$\therefore \sigma_y = 200\,N/mm^2 \sim 20\,cm$

& $\sigma_x = -100\,N/mm^2 \sim -10\,cm$

Mohr's circle is drawn as



1. Take origin & draw vertical line OY & horizontal line OX.
2. Take OA ~ σ_x ~ -10cm (10cm on negative side of origin)

   OB ~ σ_y ~ 20cm
3. Bisect AB at O' & with AB as diameter, draw circle
4. At centre O', draw an angle of $2\theta = 60°$ which cuts the circle at point C.
5. Draw CE ⊥ OX & Join OC also.
6. Measure OC, OE, CE, O'C & ∠COA

By Measurement

Length OC = 18cm & ∠COA = ϕ = 46° Ans

OE = 12.5cm

CE = 13cm

O'C = 15cm

∴ Resultant stress, σ_R = OC × scale

= 18 × 10 = 180 N/mm² Ans

& Normal stress, σ_n = OE × Scale

= 12.5 × 10

= 125 N/mm²

& Shear stress, τ = CE × Scale

= 13 × 10 = 130 N/mm²

Max shear stress, τ_{max} = O'C × Scale

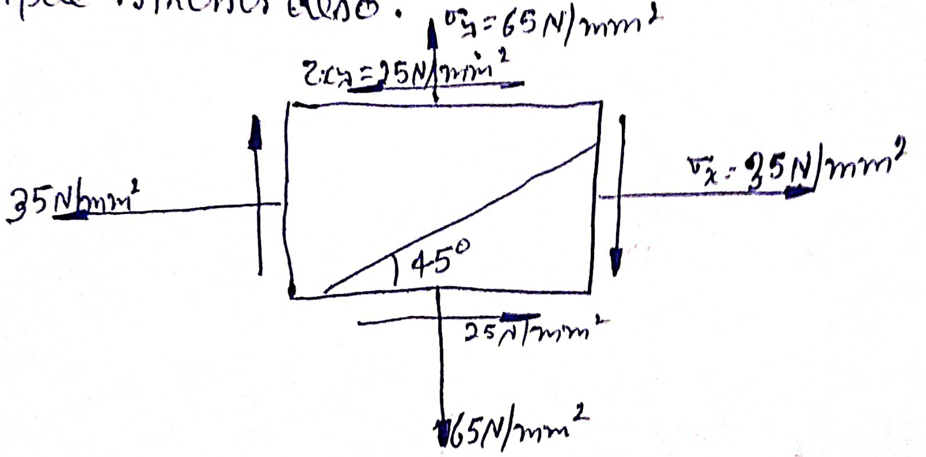
= 15 × 10

= 150 N/mm² Ans

Problem 1.12

At a certain point in a strained material, the intensities of stresses on two planes at right angle to each other are 35 N/mm² & 65 N/mm² both tensile. They are accompanied by a shear stress of magnitude 25 N/mm². Using Mohr's circle method, determine the normal, tangential ~~or~~ shear stresses & resultant stresses across the oblique plane which is 45° inclined to the axis of minimum stress. Find the location of principal planes & evaluate the principal stresses also.

Sol.



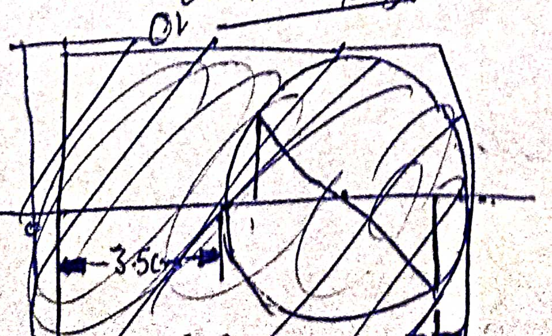
Let us assume a scale

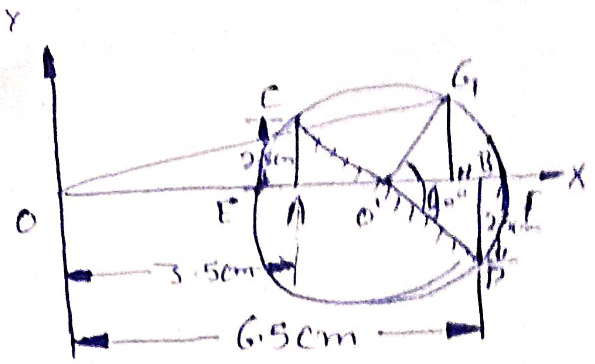
$$1\,cm \sim 10\ N/mm^2$$

$$\therefore\ \sigma_x = 35\ N/mm^2 \sim 3.5\,cm$$
$$\sigma_y = 65\ N/mm^2 \sim 6.5\,cm$$
$$\tau_{xy} = 25\ N/mm^2 \sim 2.5\,cm$$

Mohr's circle is drawn as follows:

On next page





1. Mark off $OA = \sigma_x = 3.5$cm & $OB = \sigma_y = 6.5$cm on horizontal line OX.

2. Also take $AC = \tau_{xy} = 2.5$ cm & $BD = \tau_{xy} = 2.5$cm in opposite direction.

3. Join CD which cuts the horizontal line at O'.

4. With O' as centre & O'C as radius, draw circle which cuts the horizontal line at E & F.

5. Draw an angle of $2\theta = 90°$ with CD as base line in anticlockwise direction i.e. $\angle GO'D = 2\theta = 60°$

6. Draw $GH \perp OX$ & join OG also.

7. Now measure, OE & OF, GH, OG, OH, $\angle GOO'$, $\angle BO'D$

By measurements,

Length OE = 2.08 cm

OF = 7.9cm

OH = 7.5 cm

GH = 1.5cm

OG = 7.65 cm

$\angle BO'D = 25.39°$

$\angle GOO' = \phi = 11.3°$

O'G = 5.83 cm

∴ Normal stress, σ_n = OH × scale

= 7.5 × 10 = 75 N/mm² Ans

Shear stress, τ = GH × scale

= 1.5 × 10 = 15N/mm² Ans

∴ Resultant stress, σ_R = ~~[illegible]~~ OG × scale

$= 7.65 \times 10$

$= 76.5\ N/mm^2$ Ans

Max. Principal stress, σ_1 = OF × scale $= 7.9 \times 10 = 79\ N/mm^2$

Min. Principal stress, σ_2 = OE × Scale $= 2.08 \times 10 = 20.8\ N/mm^2$ Ans

$\angle BO'D = 2\theta_1 = 25.39$

∴ Direction of principal plane $\theta_1 = \frac{25.39}{2}$

$= 12.69°$ Ans

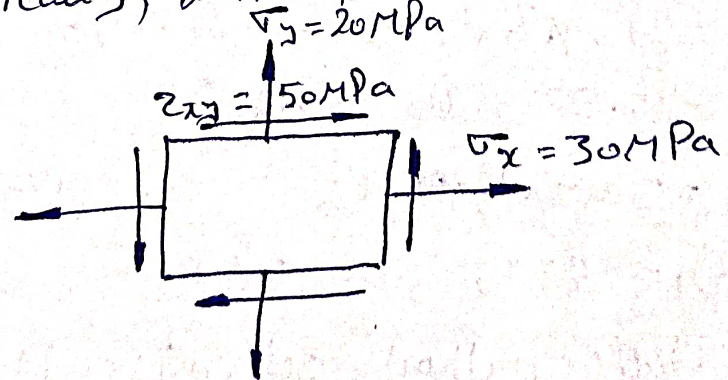
& Direction of principal plane $\theta_2 = 12.69 + 90$

$= 102.69°$ Ans

Problem 1.13

A body is subjected to two mutually perpendicular stresses of $\sigma_x = 30$ MPa & $\sigma_y = 20$ MPa both tensile. It is also subjected to a shear stress of 50MPa in anticlockwise direction. Find the location of principal planes, principal stresses & maximum shear stress (graphically) & its plane.

Sol.



NOTE:- Although, this is the case $\sigma_x > \sigma_y$, but τ_{xy} is given in opposite direction, so we have to take τ_{xy} first upward & then in downward direction

Mohr's circle is drawn as follows

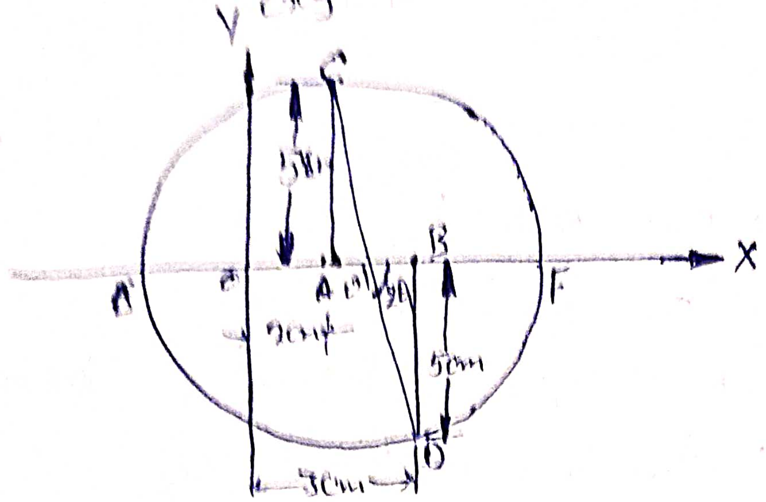
Assume scale

1cm ~ 10 MPa

∴ σ_x = 30 MPa ~ 3 cm

σ_y = 20 MPa ~ 2 cm

τ_{xy} = 50 MPa ~ 5 cm



1. Mark off OA ~ σ_x ~ 2cm & OB ~ σ_y ~ 3cm on horizontal line OX.
2. Also take AC ~ τ_{xy} ~ 5cm & BD ~ τ_{xy} ~ 5cm in opposite direction.
3. Join CD which cuts the horizontal line at O'.
4. With O' as centre & O'C as radius, draw the circle which cuts the horizontal line at E & F.
5. Now measure OE, OF, ∠BO'D, O'C

By Measurement

length OE = −2.5cm & ∠BO'D = 84°

OF = 7.5cm

O'C = 5cm

∴ Mini principal stress, σ_2 = OE×Scale = −2.5cm × Scale

= −25 MPa Ans

Max Principal stress, σ_1 = OF×Scale = 7.5 × Scale Ans

$\therefore \sigma_1 = 7.5 \times 10 MPa$
$= 75 MPa$

Max. Shear stress, $\tau_{max} = O'C \times$ Scale
$= 5 \times 10 MPa$
$= 50 MPa$ Ans

$\angle BO'D = 2\theta_1 = 84$

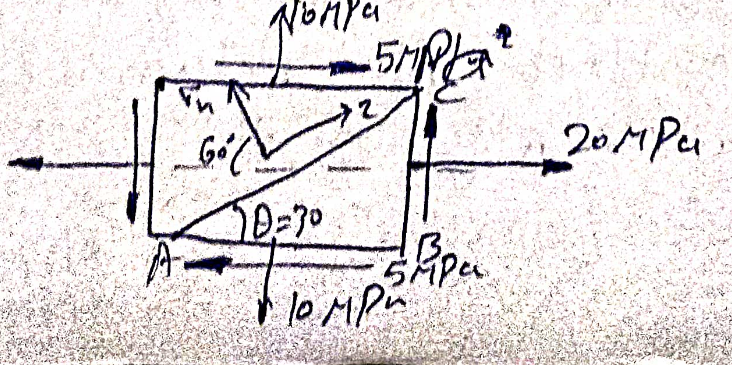
$\therefore$ Angle of principal planes $\theta_1 = 42°$
& $\theta_2 = 42° + 90° = 132°$ Ans

& Direction of plane of Max. shear stress
$\theta_3 = 42 + 45°$
$= 87°$ Ans
& $\theta_4 = 132 + 45$
$= 177°$ Ans

SOME MORE PROBLEMS

Problem 1.14 - At a point in a material, there are two normal tensile stresses of magnitude 20 MPa & 10 MPa acting mutually perpendicular to each other. There is also a positive shear stress of 5 MPa acting at that point. Determine the normal & shear stresses on a plane whose normal is inclined at 60° to 20 MPa stress. (Analytically)

Sol: -



∴ Normal stress

$\sigma_n = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy} \sin\theta \cos\theta$

$= 20 \times \sin^2 30^\circ + 10 \cos^2 30^\circ - 2 \times 5 \sin 30 \times \cos 30^\circ$

$= 20 \times \frac{1}{4} + 10 \times \frac{3}{4} - 10 \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$

$= 5 + 7.5 - 4.43 = 8.03$ MPa

Shear stress ~~from~~

$\tau = (\sigma_y - \sigma_x) \sin\theta . \cos\theta + \tau_{xy} (\cos^2\theta - \sin^2\theta)$

$= (10 - 20) \sin 30^\circ \times \cos 30^\circ + 5(\cos^2 30 - \sin^2 30)$

$= -10 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} + 5\left(\frac{3}{4} - \frac{1}{4}\right)$

$= -4.43 + 2.5$

$= -1.93$ MPa

Negative sign shows that direction of the shear stress is opposite.

Problem 1.15 :- The state of stress at a point in a material is given by two mutually perpendicular stresses as 20MPa & 10MPa & a shear stress 25 MPa. Determine the direction & magnitude of principal stresses in the material. Also locate the planes of maximum shearing stress & calculate the normal & shearing stress on these planes.

Solution :- Given $\sigma_x = 20$ MPa

$\sigma_y = 10$ MPa

$\tau_{xy} = 25$ MPa

∵ The principal stress are

$$\sigma_1, \sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}\{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2\}^{1/2}$$

$$= \frac{1}{2}(20+10) \pm \frac{1}{2}\{(20-10)^2 + 4(25)^2\}^{1/2}$$

$$= 15 \pm \frac{1}{2}(100 + 2500)^{1/2}$$

$$= 15 \pm 25.5 = 40.5, -10.5 \text{ MPa}$$ Ans

The Principal planes are given by

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 25}{20-10} = \frac{50}{10} = 5$$

$$\therefore 2\theta = 78.66°$$

$$\theta_1 = \frac{78.66}{2} = 39.33°$$ Ans

$$\& \quad \theta_2 = 39.33° + 90 = 129.33°$$ Ans

Planes of maximum shear stress are given by

$$\tan 2\theta = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$= -\left(\frac{20-10}{2 \times 25}\right) = -\frac{10}{50} = -0.2$$

$$\therefore 2\theta = -11.34$$

$$\theta_3 = -5.67°$$ Ans

$$\& \quad \theta_4 = -95.67°$$ Ans

Now, in the direction of σ_x, the angle of inclination of shear stress plane is $-5.67°$

$$\therefore \theta = -5.67°$$

∴ Normal stress on shear stress plane ($\theta = -5.67°$)

$$\sigma_n = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_y - \sigma_x)\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{1}{2}(20+10) + \frac{1}{2}(10-20)\cos(2\times -5.67) - 25\sin(2\times 5.67°)$$

$$= 15 - 5\times 0.98 + 25\times 0.197$$

$$= 15 - 4.9 + 4.925$$

$$= 15.02 \text{ MPa}$$ Ans

Shear stress on shear stress plane

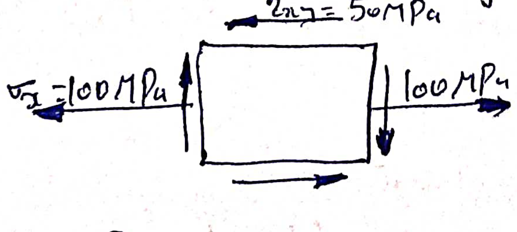
$$\tau_{max} = \frac{1}{2}(\sigma_y - \sigma_x)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= 5\times 0.197 + 25\times 0.98$$

$$= 0.98 + 24.5$$

$$= 25.48 \text{ MPa}$$ Ans

Problem 1.15 :- Find the principal stresses for the state of stress shown in fig



Solution :- Given

$\sigma_x = 100$ MPa

$\tau_{xy} = 50$ MPa

Principal stresses $\sigma_{1,2} = \frac{1}{2}(\sigma_x - \sigma_y) \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$

$$= \frac{1}{2}(100-0) \pm \frac{1}{2}\sqrt{(100-0)^2 + 4\times(50)^2}$$

$$= 50 \pm 70.71$$

∴ $\sigma_1 = 50 + 70.71 = 120.71$ MPa Ans

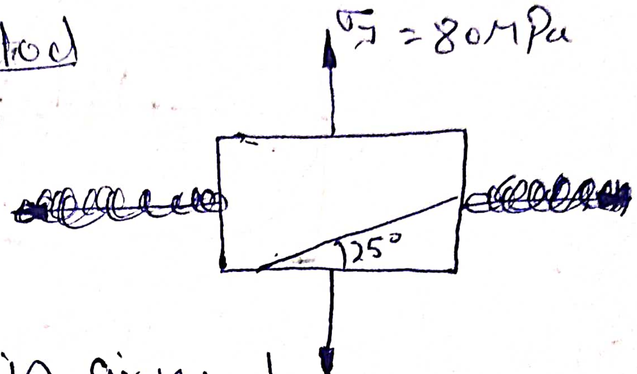
& $\sigma_2 = 50 - 70.71 = -20.71$ MPa Ans

Problem 1.16 :- Determine normal stress & tangential stress on a 25° inclined plane in a strained material which is subjected to 80MPa tensile stress as shown in fig by analytical & graphical method.

Solution :- Analytical Method

Given $\sigma_y = 80MPa$

$\theta = 25°$



Since, Normal stress is given by

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_y - \sigma_x}{2} \cos 2\theta$$

$$= \frac{80+0}{2} + \frac{80 - 0}{2} \cos 2 \times 25°$$

$$= 65.71 \text{ MPa} \quad \text{Ans}$$

& Shear stress is given by

$$\tau = \frac{1}{2}(\sigma_y - \sigma_x) \sin 2\theta$$

$$= \frac{1}{2}(80-0) \sin(2 \times 25°)$$

$$= 30.64 \text{ MPa} \quad \text{Ans}$$

Graphical Method (Mohr's Circle)

let us assume a Scale

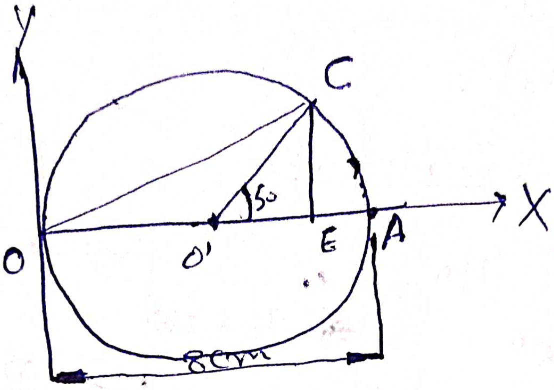
10MPa ~ 1cm

$\therefore \sigma_y = 80 MPa \sim 8cm$

1. Take origin O & draw vertical line OY & horizontal line OX.

2. Take OA equal 8cm on horizontal axis,

3. Bisect OA at O' & with O'A as radius draw circle.



4. At centre O', draw an angle of 50° which cuts the circle at point C.

5. Draw CE ⊥ OX & Join OC also

6. Measure ~~OC~~, CE, OE

By Measurements

length OE = 6.5cm

CE = 3.1 cm

~~OC~~ =

∴ Normal stress, σ_n = OE X Scale

= 6.5 X 10 MPa

= 65 MPa Ans

& Shear stress, τ = CE X Scale

= 3.1 X 10 MPa

= 31 MPa Ans

HIGHLIGHTS

1. Analytical & Graphical methods are used for finding the stresses on an oblique section.

2. When a member is subjected to direct stress (σ) in one plane i.e. under uniaxial loading, then the stresses on an oblique plane (which is inclined at an angle of θ) are given by

$$\text{Normal stress}, \sigma_n = \frac{\sigma_x}{2}(1+\cos 2\theta)$$

$$\text{Shear stress}, \tau = \frac{\sigma_x}{2}\sin 2\theta$$

$$\text{Resultant stress}, \sigma_R = \sigma_x \cos\theta$$

$$\text{Max. shear stress}, \tau_{max} = \frac{\sigma_x}{2}$$

3. For a state of uniaxial stress the maximum tangential stress occurs along planes, the normal to which make an angle of 45° & 135° with the direction of the load.

4. When a member is subjected to two like direct stresses in two mutually perpendicular directions, then the stresses on an oblique plane inclined an angle of θ with the axis of minor stress (or with the plane of major stress) are given by:

$$\text{Normal stress}, \sigma_n = \frac{\sigma_x+\sigma_y}{2} + \left(\frac{\sigma_y-\sigma_x}{2}\right)\cos 2\theta$$

$$\text{Shear stress}, \tau = \frac{1}{2}(\sigma_y-\sigma_x)\sin 2\theta$$

$$\text{Resultant stress}, \sigma_R = \left[\frac{1}{2}(\sigma_x^2+\sigma_y^2) + \frac{1}{2}(\sigma_y^2-\sigma_x^2)\cos 2\theta\right]^{1/2}$$

5. The angle made by the resultant stress with the normal of the oblique plane, is known as angle of obliquity. It is denoted by ϕ

$$\tan\phi = \frac{\tau}{\sigma_n} = \frac{\sigma_y - \sigma_x}{\sigma_x \tan\theta + \sigma_y \cot\theta}$$

6. When a member is subjected to two direct stresses in two mutually perpendicular directions, ~~those the other~~ accompanied by a shear stress (τ_{xy}) then stresses on oblique plane inclined at an angle of θ with the axis of minor stress, are given by

Normal stress, $\sigma_n = \frac{1}{2}(\sigma_y + \sigma_x) + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$

Tangential stress, $\tau = \frac{1}{2}(\sigma_y - \sigma_x)\sin 2\theta + \tau_{xy}\cos 2\theta$

7. The planes, which have no shear stress, are known as principal planes.

8. The stresses, acting on principal planes, are known as principal stresses.

9. The position of Principal planes is given by

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

10. Max. & min. Principal stresses given by

$$\sigma_1, \sigma_2 = \frac{1}{2}(\sigma_x - \sigma_y) \pm \frac{1}{2}\left\{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2\right\}^{1/2}$$

11. Max. shear stress given by

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau_{xy}^2}$$
$$= \frac{1}{2}(\sigma_1 - \sigma_2)$$

12. The plane of max shear stress is given by

$$\tan 2\theta = \frac{\sigma_y - \sigma_x}{2\tau_{xy}} = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)$$ {condition ?

The plane of max. shear stress are inclined at 45° to the plane of max. shear stress.

13. Mohr's Circle is a graphical method of finding normal, tangential & resultant stresses on an oblique plane.

14. In Mohr's Circle, on vertical axis, we take shear stress & on horizontal axis, we take, direct stresses.

EXERCISE 2

(A) THEORITICAL QUESTIONS

1. Define the following terms

(a) Principal Planes
(b) Principal Stresses
(c) Obliquity

2. Show that maximum shear stress in a body subjected to uniaxial tension or compression is half the value of applied stress.

3. Drive expression for the normal stress, tangential stress, & the resultant stress on an oblique plane when the body is subjected to direct stresses in two mutually perpendicular directions.

4) A rectangular body is subjected to direct stresses in two mutually perpendicular directions accompanied by a shear stress. Prove that the normal stress & the shear stress on an oblique plane inclined at an angle θ with the plane of major direct stress, are given by

$$\sigma_n = \frac{\sigma_y + \sigma_x}{2} + \frac{\sigma_y - \sigma_x}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

&

$$\tau = \frac{\sigma_y - \sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$$

5. A body is subjected to direct stresses in two mutually perpendicular directions accompanied by a shear stress. Draw Mohr's circle of stresses.

6. A body is subjected to direct stress in two mutually perpendicular directions. How will you determine graphically the resultant stress on an oblique plane when;
(a) The stresses are unequal & unlike
(b) The stresses are unequal & like

(B) NUMERICAL PROBLEMS

1. A cast iron block of $5 cm^2$ cross-section carries an axial compressive load of 50 kN. Calculate the magnitude of the normal & shear stresses on a plane whose normal is inclined at $30°$ to the axis of block. Also determine the maximum shear stress in the block.

[Ans $\sigma_n = -75 N/mm^2$
$\tau = -43.3 N/mm^2$
$\tau_{max} = -50 N/mm^2$]

2. A steel plate is subjected to tensile stresses of 200 MPa & 150 MPa at right angle to each other. Determine the normal & tangential stresses on a plane inclined at 60° to the 200 MPa stress. Also find the plane on which the resultant stress has maximum obliquity.

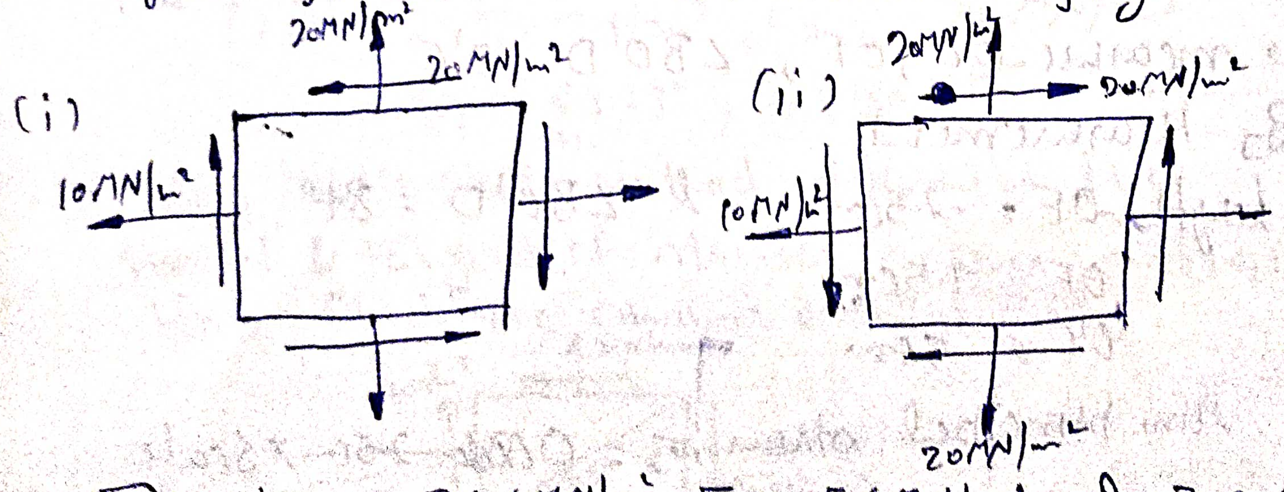
[Ans:- $\sigma_n = 187.5$ MPa, $\tau = -21.65$ MPa, $\theta = 40^\circ 43'$]

3. At a point within a body subjected to two mutually perpendicular directions, the stresses are 100 N/mm² (tensile) & 75 N/mm² (tensile). Each of the above stresses, is accompanied by a shear stress of 75 N/mm². Determine the normal, shear & resultant stresses on an oblique plane inclined at an angle of 45° with the axis of minor tensile stress.

[σ_n = 150 N/mm², τ = 25 N/mm², σ_R = 152.07 N/mm²]

Solve this problem graphically also.

4. Using Mohr's circle method, determine graphically the magnitude & direction of the planes, principal stress for the member shown in fig.



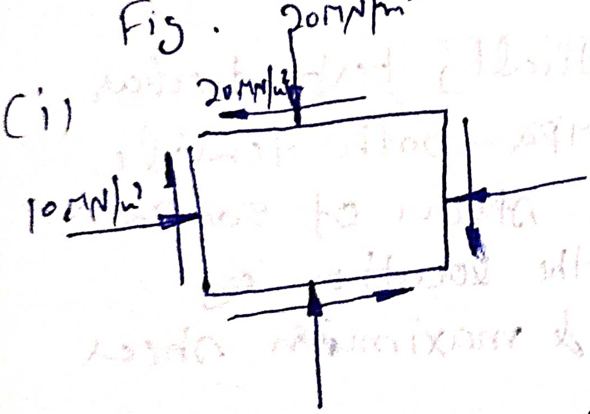
[Ans (i) $\sigma_1 = 35.6$ MN/m², $\sigma_2 = -5.6$ MN/m², $\theta_1 = 37.98^\circ$, $\theta_2 = 127.98^\circ$
(ii) $\sigma_1 = 35.6$ MN/m², $\sigma_2 = -5.6$ MN/m²; $\theta_1 = -37.98^\circ$, $\theta_2 = 52.02^\circ$]

5. An elemental cube is subjected to tensile stress of 60 N/mm² & 20 N/mm² acting on two mutually perpendicular planes accompanied by a shear stress of 20 N/mm² on these planes. Draw the Mohr's circle of stress determine the magnitude & directions of Principal stresses & also the greatest shear stress & position direction of greater shear planes

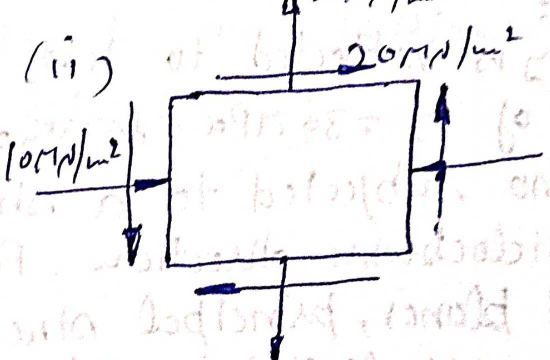
[Ans $\sigma_1 = 68.28\,N/mm^2$, $\sigma_2 = 11.72$, $\theta_1 = 25.5^\circ$, $\theta_2 = 112.5^\circ$
$\tau_{max} = 28.28\,N/mm^2$ $\theta_3 = 70.5^\circ$, $\theta_4 = 157.5^\circ$]

6. Determine graphically the magnitude & direction of Principal stresses for the elements shown in Fig.

(i)



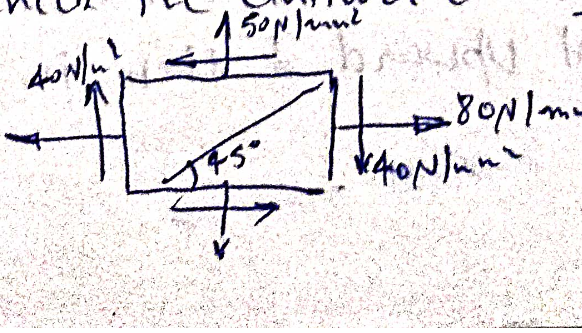
(ii)



Ans:- (i) $\sigma_1 = 5.61\,MN/m^2$, $\sigma_2 = -35.61\,MN/m^2$,
$\theta_1 = -37.98^\circ$, $\theta_2 = 52.02^\circ$

(ii) $\sigma_1 = 30\,MN/m^2$, $\sigma_2 = -20\,MN/m^2$
$\theta_1 = -53.13^\circ$, $\theta_2 = 36.87^\circ$

7. Using Mohr's circle method, determine the normal & tangential stresses across oblique plane. Check the answer analytically.



$\sigma_n = 105\,N/mm^2$
$\tau = 15\,N/mm^2$

8. A rectangular block of material is subjected to a tensile stress of 100 N/mm² on one plane & a tensile stress of 50 N/mm² on a plane at right angles, together with shear stresses of 60 N/mm² on the faces. Find:

(i) The direction of principal planes,
(ii) The magnitude of principal stresses
(iii) Magnitude of the greatest shear stress

[Ans (i) $\theta_1 = 33°41'$ $\theta_2 = 123°41'$
(ii) $\sigma_1 = 140\ N/mm^2$, $\sigma_2 = 10\ N/mm^2$
(iii) $\tau_{max} = 65\ N/mm^2$]